Instructions: Complete each of the following exercises for practice.

1. Compute the following iterated integrals.

   (a) $\displaystyle\int_{z=0}^{2}\int_{y=0}^{z^2}\int_{x=0}^{y-z} 2x - y\ dx\ dy\ dz$

   (b) $\displaystyle\int_{y=0}^{1}\int_{x=y}^{2y}\int_{z=0}^{x+y} 6xy\ dz\ dx\ dy$

   (c) $\displaystyle\int_{z=1}^{2}\int_{x=0}^{2z}\int_{y=0}^{\ln(x)} x\exp(-y)\ dy\ dx\ dz$

   (d) $\displaystyle\int_{y=0}^{1}\int_{z=0}^{1}\int_{x=0}^{\sqrt{1-z^2}} \frac{z}{y+1}\ dx\ dz\ dy$

   (e) $\displaystyle\int_{x=0}^{\pi}\int_{z=0}^{1}\int_{y=0}^{\sqrt{1-z^2}} z\sin(x)\ dy\ dz\ dx$

   (f) $\displaystyle\int_{x=0}^{1}\int_{y=0}^{1}\int_{z=0}^{2-x^2-y^2} xy\exp(z)\ dz\ dy\ dx$

2. Evaluate each triple integral $\iiint_R f(x,y,z)\ dV$.

   (a) $\iiint_R y\ dV$ where $R = \{(x,y,z) : 0 \le x \le 3, 0 \le y \le x, x-y \le z \le x+y\}$

   (b) $\iiint_R \exp(zy^{-1})\ dV$ where $R = \{(x,y,z) : 0 \le y \le 1, y \le x \le 1, 0 \le z \le xy\}$

   (c) $\iiint_R \frac{z}{x^2+z^2}\ dV$ where $R = \{(x,y,z) : 1 \le y \le 4, y \le z \le 4, 0 \le x \le z\}$

   (d) $\iiint_R \sin(y)\ dV$ where R is beneath $z = x$ and above the triangle with vertices $(0,0,0)$, $(\pi,0,0)$, and $(0,\pi,0)$

   (e) $\iiint_R 6xy\ dV$ where R is below $z = 1+x+y$ and above xy-region bounded by $y = \sqrt{x}$, $y = 0$, and $x = 1$

   (f) $\iiint_R (x-y)\ dV$ where R is bounded by surfaces $z = x^2-1$, $z = 1-x^2$, $y = 0$, and $y = 2$

   (g) $\iiint_R y^2\ dV$ where R is the solid tetrahedron with vertices $(0,0,0)$, $(2,0,0)$, $(0,2,0)$, and $(0,0,2)$

   (h) $\iiint_R xz\ dV$ where R is the solid tetrahedron with vertices $(0,0,0)$, $(1,0,1)$, $(0,1,1)$, and $(0,0,1)$

   (i) $\iiint_R x\ dV$ where R is the region bounded by the surfaces $x = 4y^2+4z^2$ and $x = 4$

   (j) $\iiint_R z\ dV$ where R is the region bounded by $y^2+z^2 = 9$, $x = 0$, $y = 3x$, and $z = 0$ in the first octant

3. Compute the (signed) Jacobian of the transformation.

   (a) $\begin{cases} x &= uv \\ y &= vw \\ z &= wu \end{cases}$

   (b) $\begin{cases} x &= u+vw \\ y &= v+wu \\ z &= w+uv \end{cases}$

4. Compute the integral $\iiint_R f(x,y,z)\ dV$ by making a coordinate change to cylindrical coordinates.

   (a) $f(x,y,z) = \sqrt{x^2+y^2}$ and R is the region bounded by $x^2+y^2 = 16$, $z = -5$, and $z = 4$

   (b) $f(x,y,z) = z$ and R is the region bounded by $z = x^2+y^2$ and $z = 4$

   (c) $f(x,y,z) = x+y+z$ and R is the region in the first octant bounded by $z = 4-x^2-y^2$

   (d) $f(x,y,z) = x-y$ and R is between cylinders $x^2+y^2 = 1$ and $x^2+y^2 = 16$, above $z = 0$, and below $z = y+4$

   (e) $f(x,y,z) = x^2$ and R is the region inside $x^2+y^2 = 1$, above $z = 0$, and below $z^2 = 4x^2+4y^2$

5. Compute the integral $\iiint_R f(x,y,z)\ dV$ by making a coordinate change to spherical coordinates.

   (a) $f(x,y,z) = (x^2+y^2+z^2)^2$ and R is the ball of radius 5 about the origin

(b) $f(x, y, z) = y^2z^2$ and R is the region above the cone $\phi = \frac{\pi}{3}$ and inside the sphere $\rho = 1$

(c) $f(x, y, z) = x^2 + y^2$ and R is the region satisfying $4 \leq x^2 + y^2 + z^2 \leq 9$

(d) $f(x, y, z) = y^2$ and R is the solid hemisphere of $x^2 + y^2 + z^2 \leq 9$ with $y \geq 0$

(e) $f(x, y, z) = x\exp(x^2 + y^2 + z^2)$ and R is the portion of the unit ball $x^2 + y^2 + z^2 \leq 1$ in the first octant

(f) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ and R is the region between the spheres $x^2 + y^2 + z^2 = 1$ and $x^2 + y^2 + z^2 = 4$ and above the cone $z = \sqrt{x^2 + y^2}$

6. Compute the volume of the solid R...
   (a) within both the cylinder $x^2 + y^2 = 1$ and the sphere $x^2 + y^2 + z^2 = 4$
   (b) in both the sphere $x^2 + y^2 + z^2 = 2$ and the cone $z = \sqrt{x^2 + y^2}$
   (c) between the paraboloid $z = x^2 + y^2$ and the sphere $x^2 + y^2 + z^2 = 2$
   (d) between the paraboloid $z = 24 - x^2 - y^2$ and the cone $z = \sqrt{x^2 + y^2}$
   (e) cut out by the cylinder $r = a\cos(\theta)$ and sphere of radius $a > 0$ centered at the origin
   (f) lying above the cone $\phi = \frac{\pi}{3}$ and below the sphere $\rho = 4\cos(\phi)$
   (g) within the sphere $x^2 + y^2 + z^2 = 4$, above the xy-plane, and below the cone $z = \sqrt{x^2 + y^2}$
   (h) above the cone $z = \sqrt{x^2 + y^2}$ and within the sphere $x^2 + y^2 + z^2 = 1$

7. Compute the center of mass of the solid bounded by the paraboloid $z = 4x^2 + 4y^2$ and the plane $z = a$ for $a > 0$, supposing the solid has constant density K.

8. Compute the mass of the ball B of radius r about the origin if the density at a point is proportional to its distance from the z-axis.

9. Compute the average distance from a point in a ball of radius r to its center.